Copy 6
RM A55G08

NACA

RESEARCH MEMORANDUM

COMPARISON BETWEEN ANALYTICAL AND WIND-TUNNEL RESULTS ON FLUTTER OF SEVERAL LOW-ASPECT-RATIO, HIGH-DENSITY, UNSWEPT WINGS AT HIGH SUBSONIC SPEEDS AND ZERO ANGLE OF ATTACK

By Robert W. Warner

Ames Aeronautical Laboratory
Moffett Field, Calif.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WASHINGTON

September 20, 1955

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

RESEARCH MEMORANDUM

COMPARISON BETWEEN ANALYTICAL AND WIND-TUNNEL RESULTS ON FLUTTER OF SEVERAL LOW-ASPECT-RATIO, HIGH-DENSITY, UNSWEPT WINGS AT HIGH SUBSONIC SPEEDS AND ZERO ANGLE OF ATTACK

By Robert W. Warner

SUMMARY

Experimental flutter Mach numbers for several solid, thin, rectangular cantilever wings with uniform section properties, low aspect ratio, and high relative density have been estimated from the results of previous tests at zero angle of attack. These experimental values are considered estimates, rather than determinations, in the high subsonic speed range because in that range the amplitude criterion used for the flutter Mach numbers, although carefully chosen and consistently applied, was necessarily arbitrary. The experimental estimates are compared with a so-called "standard" analysis and what is herein termed a "refined" analysis. The standard analysis was unconservative relative to experiment for the wings of highest relative density. The refined analysis contained approximate corrections for compressibility and finite span effects which improved the agreement between analysis and experiment.

INTRODUCTION

The basic purpose of the present report is the comparison of two types of flutter analysis with wind-tunnel results which indicate low-angle-of-attack, bending-torsion flutter. These results were obtained during tests reported in references 1 and 2. Reference 1 is a flutter report. Reference 2 is a static-data report, but during the tests reported in reference 2 hitherto unpublished flutter data were recorded as a by-product.

The wings considered herein are solid, thin, rectangular, and cantilever with uniform section properties, low aspect ratio, and high relative density. The estimated flutter Mach numbers are generally in the high subsonic speed range. The test Mach numbers ranged from 0.4 to 1.10 with corresponding Reynolds numbers from 1.25 to 2.05 million. The tests, described in references 1 and 2, were performed on the transonic bump of the Ames 16-foot high-speed wind tunnel.

Two types of flutter analysis, both employing the first-bending and first-torsional modal distributions along the span, are used for comparison with the experimental flutter Mach numbers. One is the so-called "standard" analysis. This occasionally, as in the present case, employs modal distributions but often does not (ref. 3). The other type is referred to herein as the "refined" analysis because it represents a refinement relative to the standard analysis. No implication of absolute refinement is intended.

In the standard analysis, two-dimensional air forces for incompressible flow are used without consideration of aerodynamic effects of span. In the refined analysis, two-dimensional air forces for compressible flow (tabulated in ref. 4) are employed with air-force magnitudes corrected in such a manner that the load distribution would be elliptic if the wing were rigid. In all other respects the two types of analysis are similar in principle.

A great deal of work has already been done on the comparison of flutter analysis with experiment, as indicated in references 5 to 13. In general, the emphasis has been placed on ascertaining whether the standard analysis is conservative relative to experiment. A generalization which can be inferred from the data in references 5 to 13 is that standard analysis has always been conservative for unswept wings tested at high subsonic speeds and low angles of attack, provided the relative density is greater than 40 and the structural aspect ratio (defined in the list of symbols) is less than 9. Since the wings of references 1 and 2 are in this category, the present comparison of experimental and standard analytical flutter Mach numbers is important as a check on the generalization as to conservatism.

A second type of comparison is also of importance. Regardless of conservatism, how close is the standard or refined analytical flutter Mach number to the experimental flutter Mach number?

The uncertainties in the analytical and experimental flutter Mach numbers are also considered. Since the vibrations reported at zero angle of attack in reference 1 occurred over a wide Mach number range, it has been difficult to determine the flutter Mach numbers for the wings from that reference. Hence, the experimental flutter Mach numbers are considered estimates rather than determinations. The method of estimation is given detailed consideration herein. Further illustration of the difficulty of experimental flutter Mach number estimation in the transonic speed range can be found in reference 14.

SYMBOLS

A	full-span structural aspect ratio (Fuselage, when present, is not included in span.)
M	Mach number of flutter in wind tunnel
M_o	Mach number of flutter according to standard analysis
M_r	Mach number of flutter according to refined analysis
V_a	fictitious flutter velocity, assumed in analysis, ft/sec
a	distance of elastic axis aft of midchord, in wing semichords
b	wing semichord, ft
c	speed of sound in test section at flutter Mach number, ft/sec
k_a	fictitious reduced frequency, assumed in analysis, $\frac{\omega_a b}{V_a}$
m	wing weight per unit span, lb/ft
r_α	radius of gyration of wing section per unit span, in wing semichords
x_α	distance of section center of gravity aft of elastic axis, in wing semichords
δ	displacement ratio, the maximum wing thickness in wing chords or the double amplitude of vertical motion in wing chords, whichever is larger
ρ	air density in test section at flutter Mach number, lb/cu ft
μ	relative density of wing, $\frac{m}{\pi\rho b^2}$
ω	frequency of flutter in wind tunnel, radians/sec
ω_a	fictitious flutter frequency corresponding to V_a and k_a, radians/sec
ω_o	frequency of flutter according to standard analysis, radians/sec
ω_r	frequency of flutter according to refined analysis, radians/sec
ω_α	first natural torsional frequency, radians/sec
ω_h	first natural bending frequency, radians/sec

ANALYTICAL ESTIMATION OF FLUTTER CHARACTERISTICS

Description of the Two Types of Analysis

Aerodynamic parameters used in the analyses.- Two-dimensional strip theory for incompressible flow is used in the standard analysis (ref. 3). In the refined analysis two-dimensional air forces for compressible flow (tabulated in ref. 4) are employed and are modified so that the load distribution would be elliptic if the wing were rigid. For the standard analysis, then, air forces vary according to the modal distribution along the span. For the refined analysis, air forces vary according to the product of the modal distribution and the elliptic loading. The elliptic finite span correction does not affect the phase of the air forces.

Structural parameters used in the analyses.- In both types of analysis the structural damping is taken to be zero, a good assumption for solid metal wings. Section mass distributions, center-of-gravity positions, and radii of gyration are determined analytically, and the elastic-axis locations are measured. The mode shapes used for all wings and the structural frequencies used for the wings with NACA 63A-002 section are those calculated by uniform beam theory for a fixed-root cantilever beam. For the wings from reference 1 (NACA 64A-002 section) the measured structural frequencies are used.

Analytical techniques.- In both the refined and the standard analyses, the number of degrees of freedom considered is restricted to two, first bending and first torsion, and these are modal distributions along the span. The two structural frequencies, although actually known, are taken as the two variables in the flutter equation.

For the standard analysis (with Mach number always assumed to be zero for the air forces) the flutter equations can be solved when a value of the reduced frequency is selected. The solutions for the two structural frequencies are expressed in terms of the flutter speed. Thus a grid of curves having reduced frequency k_a and flutter speed V_a as parameters can be put on plots with the structural frequencies as axes. The known values of the structural frequencies then give the actual analytical reduced frequency and flutter speed, and from them the analytical flutter frequency can be found.

For the refined analysis, however, a Mach number, as well as a reduced frequency, must be assumed for the air forces before the flutter equations can be solved. The calculated flutter Mach number corresponding to the assumed Mach number is then found as in the standard analysis, with the flutter speeds, V_a, selected to bracket the assumed Mach number. The calculation is repeated with different assumed Mach numbers until one is found which agrees closely with its resulting calculated flutter Mach number. The actual analytical flutter Mach number is then taken as the average of the final assumed and calculated Mach numbers.

Figure 1 shows, for the 2-percent-thick aluminum wing of aspect ratio 3 described in table I, the parametric flutter curves for the standard analysis and for the final iteration in the refined analysis.

Possible Sources of Error in Analytical Flutter Mach Number

Possible errors due to the aerodynamic parameters.- In the discussion of the aerodynamics only the refined analysis is considered since the standard analysis is merely expected to employ "standard" air forces, not close approximations of actual air forces. There are two general categories of error in the aerodynamics of the refined analysis: the linearization of the air forces and the approximation of the finite-span effects. The significance of the linearization is considered first.

The linear aerodynamic theory applies, of course, only in the ranges of Mach number, M, reduced frequency, k, aspect ratio, A, and displacement ratio, δ, in which there is no flow separation. (See the list of symbols for the definitions of these terms.) For an oscillatory thin wing of finite span at any Mach number, Miles (ref. 15) states the necessary conditions for linearization. All of the following must be satisfied:

$$\delta,\ M\delta,\ k\delta,\ kM\delta \ll 1 \tag{1}$$

and at least one of the following:

$$\left.\begin{aligned} |M-1| &\gg \delta^{2/3} \\ k &\gg \delta^{2/3} \\ A\delta^{1/3} &\ll 1 \end{aligned}\right\} \tag{2}$$

Note that Miles confines δ to the thickness ratio but his basic reference, reference 16 in the present report, defines δ as used herein.

Since the refined analysis covers the ranges $0 < M \leqq 1$ and $0 < k < 1$, conditions (1) are everywhere satisfied for reasonably mild oscillations of the present thin wings. Such M and k ranges, however, mean that the first two of conditions (2) are not everywhere satisfied. Hence, with an exception noted later, justification for linearization is expected from the inequality:

$$A\delta^{1/3} \ll 1 \tag{3}$$

From the theoretical viewpoint, the inequality (3) applies to the steady or the oscillatory case, with δ including only the thickness ratio in the steady case.

Now, through the analysis of experimental results, McDevitt (ref. 17) has extended condition (3) for the steady case (rectangular wings) to the following:

$$A\delta^{1/3} \leqq 1 \tag{4}$$

The fact that the theoretically determined inequality (3) applies to both the static and the oscillatory cases suggests that the experimental boundary (4) can be extended from the static to the oscillatory case. This extension requires experimental verification, of course. In the absence of such verification, however, it is assumed that linear theory is sufficiently accurate for the present oscillatory wings if those wings fall in the region defined by boundary (4).

In the application of condition (4) to the present wings, δ is first considered as the thickness ratio. If a wing satisfies condition (4) with a margin, then oscillations with an amplitude ratio higher than the thickness ratio by an amount sufficient to remove the margin can be analyzed with linear theory even at $M = 1$ and $k \cong 0$. The present restrictions do not require that linear oscillatory air forces can exist when the steady air forces are nonlinear. (In this connection it is interesting to note that on page 30 of reference 18, Mollo-Christensen and Lewis conclude for the wings they tested "that for very low amplitudes of oscillation, the linear unsteady effects can be superimposed upon the nonlinear thickness effects.")

On the basis of the present criterion, the linear theory is sufficiently accurate at small amplitudes for all wings in table I except the aspect-ratio-4 and -6 wings. Actually, the aspect-ratio-4 wing has $A\delta^{1/3} = 1.09$, which might be considered borderline; and the aspect-ratio-6 wing is expected to flutter at a sufficiently low Mach number to permit linearization on the basis of the first of conditions (2). Altogether, little error is anticipated at low amplitudes as a result of the application of linearized theory, provided the boundary (4) actually can be applied to the oscillatory case.

A greater likelihood of significant error in the air forces used for the refined analysis appears to lie in the second category, the approximation of the finite-span effects. There is no theoretical justification for superposing a finite-span correction on a Mach number correction. While the present approach probably gives a good approximation as to the effect of finite span on air-force magnitude, no correction is included as to phase. This is certain to cause some error, which is felt to be significant but not unduly large. The precise magnitude of the error cannot be evaluated since air forces on an oscillating and deforming rectangular wing at high subsonic speeds have not been tabulated.

Possible errors due to the structural parameters.- It is believed that the only possible sources of error worth considering among the structural parameters are the neglect of chordwise bending and the use of analytical structural frequencies for the wings of NACA 63A-002 section. Concerning chordwise bending, the flutter movies showed no distinct trace of it. Its neglect is also justified to some extent by the fact that the use of fixed-root beam theory duplicated the measured structural frequencies of reference 1. Such agreement also justifies the use of analytical structural frequencies for the wings of NACA 63A-002 section, particularly since those wings would tend to have effectively fixed roots during a brief flutter observation (the root fixity is discussed more fully later). Apparently there is little likelihood of error due to the structural parameters.

Possible errors due to the analytical techniques.- In both the standard and the refined analyses, there is some question as to whether the first bending and first torsion modes are sufficient to describe the flutter motion. It seems they are for the present wings, however, since a preliminary analysis which also included the second bending mode showed that that mode contributed essentially nothing to the theoretical flutter shapes and did not change the flutter speeds. Hence, the second bending mode was ignored in subsequent analyses.

If the standard analysis is actually to be a reference type of analysis, then variations from that "standard" may be regarded as errors. Only one such variation is present in standard analysis as generally applied to unswept wings of the aspect ratio and relative density range under consideration. That is the use or nonuse of modal distributions along the span (see refs. 5 to 13). Preliminary calculations indicated that when first bending and first torsion are the significant structural frequencies, the analytical flutter speeds are essentially the same with and without modal functions. Hence the use of modal functions for the present standard analyses does not represent a significant deviation from any previous standard analyses.

In the refined analysis there may be a small error arising from the averaging of calculated and assumed Mach numbers to give the analytical flutter Mach numbers. Since the iterations were continued until the calculated and assumed Mach numbers differed by an increment of less than 0.05 this error is not considered significant. In general, it is believed that the analytical techniques contribute little or no error to the analytical estimation of the flutter characteristics.

EXPERIMENTAL ESTIMATION OF FLUTTER CHARACTERISTICS

Tests Including Supercritical Flutter Mach Numbers

Models.- The models for which supercritical flutter Mach numbers are available are described in reference 1. They were solid aluminum or steel, rectangular, and cantilever with no fuselage or external stores. All sections were 2 percent thick and symmetrical.

The models from reference 1 considered at present are the 64A-002 models listed in table I. Relative densities (μ) ranged from 56.5 to 186.6, and structural aspect ratios varied from 2-2/3 to 4.

Test procedure.- In the tests reported in reference 1, careful attention was given to attaining a rigid root fixity. The wings were attached to the massive transonic bump of the 16-foot wind tunnel with clamps contoured to the wing profiles (see fig. 2 for tunnel test section). The clamps exerted 800 pounds of force from the action of an air cylinder. The effectiveness of the clamps was checked by retesting observed flutter conditions with the clamps rigidly bolted, and the observed flutter conditions did not change.

The procedure consisted essentially of the visual observation of any vibration that occurred for each combination of aspect ratio, Mach number, and angle of attack (only angles of attack within $1/2^{\circ}$ of zero are considered in the present report). Where possible, frequencies were determined by comparing on an oscilloscope the signals from an audio oscillator and from a vibration pickup. In a few of the cases where the oscilloscope indicated no unique frequency, high-speed motion pictures were available and enabled a definite frequency to be determined.

Method of experimental flutter Mach number estimation.- As stated in reference 1, the vibrations at low angles of attack were hard to define (contrary to the stall flutter). The difficulty is evident in figures 3 and 4, taken from reference 1, where vibrations of limited amplitude are reported over a wide range of Mach numbers at zero angle of attack. It is not valid to assume that the flutter Mach number is the lowest Mach number where any low-intensity vibrations were observed. Such vibrations could be forced by the smallest amount of tunnel-wall vibration or airflow roughness, provided the positive damping has been sharply reduced by the air forces. On the other hand, it cannot be said that flutter is not present at a given Mach number simply because the amplitude is relatively low inasmuch as aerodynamic nonlinearities could prevent destructive oscillations, particularly when the Mach number is near or in the subsonic but supercritical speed range of the model.

Hence a criterion was required to fix the degree of vibration intensity that could be regarded as the beginning of low-angle-of-attack flutter. Since the criterion must be applied to the data of reference 1, certain of the figures from that reference which show points of "intermittent flutter" and "steady flutter" on plots of angle of attack versus Mach number are reproduced in the present report as figures 3 to 6. Figure 5 is illustrative of the figures from reference 1 in which there is no subsonic flutter at zero angle of attack. Figure 6 is included to show the only case of apparent subsonic flutter at zero angle of attack which was rejected, for reasons given below. Hence figures 3 and 4 are the only ones from which flutter Mach numbers were obtained for the wings from reference 1.

The flutter Mach number is defined as the lowest Mach number at which the damping goes to zero. In the present criterion it is assumed that the damping reached zero at those Mach numbers where "intermittent flutter" or "steady flutter" at zero angle of attack reached such an intensity that the observers felt they could not safely raise the angle of attack above zero. Thus in figures 3 to 6 the flutter Mach number is the lowest Mach number at zero angle of attack for which a cross is superimposed on a circle or a square.

As an example of the application of this criterion, the flutter Mach numbers from figure 3 are 1.06 for $A = 3.00$, 0.98 for $A = 3.33$, and 0.94 for $A = 4.00$. Actually, results for the $A = 3.00$ wing were not used because the present report is not concerned with supersonic flutter. The consistent variation of flutter Mach number with aspect ratio that is indicated for these wings could have been maintained by the $A = 3.67$ wing if the experiment for that wing had included Mach numbers higher than 0.94.

As mentioned previously, one case where the above criterion was satisfied at subsonic speeds was rejected; that case was at $M = 0.85$ for the $A = 5.00$ wing of figure 6. The data for this wing were rejected because there was no sequence, with varying aspect ratio, of flutter Mach numbers satisfying the criterion. It is possible that the violent vibrations at angles of attack slightly above zero were caused primarily by aerodynamic disturbances resulting from the spanwise-running slots on these particular wings. This possibility is strengthened by the lack of such vibrations for the corresponding unslotted wing (fig. 5).

The criterion used has two advantages for present purposes: First, it is directly related to the data in reference 1, which are felt to be repeatable. Second, since it is applied consistently, it increases the probability that all flutter Mach number estimates are in the same part of the range of uncertainty.

Sources of uncertainty in the estimation of experimental flutter Mach numbers.- The sources of uncertainty in the test procedure which must be qualitatively evaluated are buffeting and wind-tunnel resonance.

Buffeting is no problem because the tests reported in reference 19 indicate that the buffet force is negligible at zero angle of attack for the very thin wings considered herein. Wind-tunnel resonance frequencies were not calculated because of complications due to the odd tunnel cross section (see ref. 20 and fig. 2 of the present report). Even at resonance frequencies, it is felt that resonance effects would be small because (1) the wing was small relative to the test section, and (2) the reflections would be dispersed to some extent.

The fact that the criterion for flutter Mach numbers was arbitrary represents the principal uncertainty, and a major one, in the estimation. One thing, however, is believed to be certain, namely, that at the Mach number established by the criterion, the wing was fluttering. It is not likely that a vibration sufficiently violent to make an observer unwilling to raise the angle of attack above zero could be forced by a small amount of flow roughness or wind-tunnel vibration as long as positive damping is present in any significant quantity. The only remaining cause of vibration for the present wings in the present wind tunnel is flutter. Hence, the criterion can be in error only insofar as it determines too high a flutter Mach number. The degree of this uncertainty is an unknown quantity. It is felt to be significant but not unduly large.

Tests Resulting in Subcritical Flutter Mach Numbers

Models.- The two models in table I with the NACA 63A-002 section, which were tested with the wings of reference 2 but not reported therein, both fluttered in the subcritical speed range. These models were similar to those from reference 1 with the following exceptions: Relative densities (μ) were 43.1 and 46.0 with both models made of solid aluminum alloy. Structural aspect ratios were 4 and 6.

Test procedure.- In the tests reported in reference 2, the models were rigidly attached to a strain-gage balance in the transonic bump. (See fig. 2 for tunnel test section.) Since the balance was very heavy, it is felt that the model roots were effectively fixed, at least for the brief time interval required for a flutter observation.

The procedure consisted of the visual observation of any vibration that occurred for each combination of aspect ratio, Mach number, and angle of attack (only angles of attack within $1/2^{\circ}$ of zero are considered in the present report).

Method of experimental flutter Mach number estimation.- The flutter Mach numbers were simply selected as those where the observers first saw violent vibrations at zero angle of attack. The only wings which vibrated violently but are excluded from the present report are those which did so only at an angle of attack well above zero.

Sources of uncertainty in the estimation of experimental flutter Mach numbers.- Again there is little uncertainty resulting from the procedure. Buffeting and tunnel resonance are considered unimportant for the same reasons given for the wings of reference 1 (NACA 64A-002 section). Since violent vibrations developed rapidly with increasing Mach number at definitely subcritical Mach numbers, the flutter Mach number estimates seem essentially free from uncertainty for the wings with the NACA 63A-002 section.

RESULTS AND DISCUSSION

Presentation of Results

The comparison between experimental and analytical flutter Mach numbers is presented in table I and figure 7. The analysis predicted no subsonic flutter for the wings which did not flutter subsonically. As stated previously, frequency data for the wings of reference 1 were limited by difficulties in oscilloscope reading and a shortage of high-speed motion pictures. Reference 2 is a static-data report and frequencies were not measured during the tests reported therein. Hence, experimental flutter frequencies are given only for three of the wings from reference 1. Figure 8 shows a cycle of motion from the high-speed movies of the aspect-ratio-3 aluminum wings and is illustrative of oscillation amplitudes well above the estimated flutter Mach number.

Conservatism of Standard Analysis Relative to Experiment

In this section the concern is not whether the standard analysis gives flutter Mach numbers which are close to those of experiment but rather whether the standard analysis is conservative relative to experiment. It can be seen from figure 7 and from the M/M_o column of table I that the standard analysis was conservative for the aluminum wings and unconservative for the steel wings. This result is more likely to be a relative density effect than a Mach number effect since one of the aluminum wings had an estimated flutter Mach number as high as those of the steel wings. Also, analytical flutter Mach numbers by the standard analysis are higher than those by the refined analysis only for the steel wings.

As pointed out in the Introduction, previous tests of similar wings (unswept, relative density greater than 40, structural aspect ratio less than 9) in the same speed and angle-of-attack range showed the standard analysis always to be conservative. The present steel wings contradict this trend. It should be noted, however, that none of the wings used in establishing the trend had relative densities as high as the present steel wings.

Closeness of Analytical and Experimental Results

In this section the concern is with the closeness of analytical and experimental results rather than with conservatism. Frequency comparisons are excluded because of insufficient data.

The M/M_r column of table I shows that the refined analysis gave flutter Mach numbers within 5 percent of the experimental flutter Mach numbers for all but one of the wings and within 10 percent for all the wings. The M/M_o column shows that standard analysis gave only two flutter Mach numbers within 5 percent of the corresponding experimental values and that for two of the six wings the difference exceeded 10 percent. The absolute comparisons are most easily seen in figure 7.

Interpretation of Analytical and Experimental Uncertainties

The standard analysis is probably sufficiently accurate as a reference type of analysis. As a means of flutter Mach number estimation, however, it suffers from the fact that "standard" air forces are not intended to be realistic for the present wings.

The refined analysis should be better as an actual means of estimation. The discussion of possible errors in the refined analysis reduced the significant possibilities to the lack of phase correction in the finite-span approximation. The magnitude of this error cannot be rigorously evaluated but is felt to be not unduly large.

The examination of experimental uncertainties developed the following: (1) that the estimates for the two wings which vibrated violently in the subcritical speed range (those with the NACA 63A-002 section) are probably essentially accurate, and (2) that the estimates for the four wings with violent vibrations largely in or near the supercritical speed range (those from ref. 1) probably give the upper limits for the actual flutter Mach numbers but still involve uncertainties of unknown magnitude. Although these uncertainties are not felt to be unduly large, the experimental flutter Mach number estimations for the wings of reference 1 cannot be regarded as determinations.

CONCLUDING REMARKS

Experimental flutter Mach numbers at zero angle of attack have been estimated from the results of tests reported in references 1 and 2. The results at high subsonic speeds from reference 1 are considered estimates, rather than determinations, because at those speeds the amplitude criterion used for the flutter Mach numbers, although carefully chosen and consistently applied, was necessarily arbitrary. The experimental values have been compared with a "standard" analysis and what has been called a "refined" analysis. The following are the principal concluding remarks:

1. The standard analysis was conservative relative to experiment for the aluminum wings and unconservative for the steel wings.

2. The refined analysis gave flutter Mach numbers within 5 percent of the experimental flutter Mach numbers for all but one of the wings and within 10 percent for all the wings. For several of the wings, standard analysis gave a much wider disagreement.

Ames Aeronautical Laboratory
National Advisory Committee for Aeronautics
Moffett Field, Calif., July 8, 1955

REFERENCES

1. Herrera, Raymond, and Barnes, Robert H.: An Experimental Investigation of the Flutter of Several Wings of Varying Aspect Ratio, Density, and Thickness Ratio at Mach Numbers from 0.60 to 1.10. NACA RM A54A29, 1954.

2. Nelson, Warren H., and McDevitt, John B.: The Transonic Characteristics of 22 Rectangular, Symmetrical Wing Models of Varying Aspect Ratio and Thickness. NACA TN 3501, 1951. (Formerly NACA RM A51A12)

3. Scanlan, Robert H., and Rosenbaum, Robert: Introduction to the Study of Aircraft Vibration and Flutter. The Macmillan Company, 1951, pp. 201-220.

4. Jordan, P. F.: Aerodynamic Flutter Coefficients for Subsonic, Sonic and Supersonic Flow (Linear Two-Dimensional Theory). Rep. Structures 141, British R. A. E., Apr. 1953.

5. Barmby, J. G., Cunningham, H. J., and Garrick, I. E.: Study of Effects of Sweep on the Flutter of Cantilever Wings. NACA Rep. 1014, 1951.

6. Clevenson, S. A., and Lauten, William T., Jr.: Flutter Investigation in the Transonic Range of Six Airfoils Attached to Three Freely Falling Bodies. NACA RM L7K17, 1948.

7. Lauten, William T., Jr., and Nelson, Herbert C.: Results of Two Free-Fall Experiments on Flutter of Thin Unswept Wings in the Transonic Speed Range. NACA RM L51C08, 1951.

8. Bursnall, William J.: Initial Flutter Tests in the Langley Transonic Blowdown Tunnel and Comparison With Free-Flight Flutter Results. NACA RM L52K14, 1953.

9. Barmby, J. G., and Teitelbaum, J. M.: Initial Flight Tests of the NACA FR-2, a High-Velocity Rocket-Propelled Vehicle for Transonic Flutter Research. NACA RM L7J20, 1948.

10. Lundstrom, Reginald R., Lauten, William T., Jr., and Angle, Ellwyn E.: Transonic-Flutter Investigation of Wings Attached to Two Low-Acceleration Rocket-Propelled Vehicles. NACA RM L8I30, 1948.

11. Widmayer, E., Jr., Lauten, W. T., Jr., and Clevenson, S. A.: Experimental Investigation of the Effect of Aspect Ratio and Mach Number on the Flutter of Cantilever Wings. NACA RM L50C15a, 1950.

12. Jones, George W., Jr., and DuBose, Hugh C.: Investigation of Wing Flutter at Transonic Speeds for Six Systematically Varied Wing Plan Forms. NACA RM L53G10a, 1953.

13. Molyneux, W. G., Ruddlesden, F., and Cutt, P. J.: Flutter Tests on Unswept Wings Using Ground-Launched Rockets. Rep. Structures 118, British R. A. E., Nov. 1951.

14. Land, Norman S., and Abbott, Frank T., Jr.: Transonic Flutter Investigation of a Fighter-Type Airplane Wing Model. NACA RM L55B16, 1955.

15. Miles, John W.: On Linearized Theory. Readers' Forum, Jour. Aero. Sci., vol. 20, no. 1, Jan. 1953, p. 64.

16. Lin, C. C., Reissner, E., and Tsien, H. S.: On Two-Dimensional Non-Steady Motion of a Slender Body in a Compressible Fluid. Jour. Math. and Physics, vol. 27, no. 3, Oct. 1948, pp. 220-231.

17. McDevitt, John B.: A Correlation by Means of the Transonic Similarity Rules of the Experimentally Determined Characteristics of 22 Rectangular Wings of Symmetrical Profile. NACA RM A51L17b, 1952.

18. Mollo-Christensen, Erik L., and Lewis, S. Dean: An Exploratory Investigation of Unsteady Transonic Flow. Part II, Experimental Results. TACP Rep. 6, Aero. Eng. Dept., M.I.T., Aug. 1954.

19. Coe, Charles F., and Mellenthin, Jack A.: Buffeting Forces on Two-Dimensional Airfoils as Affected by Thickness and Thickness Distribution. NACA RM A53K24, 1954.

20. Woolston, Donald S., and Runyan, Harry L.: Some Considerations on the Air Forces on a Wing Oscillating Between Two Walls for Subsonic Compressible Flow. Jour. Aero. Sci., vol. 22, no. 1, Jan. 1955, pp. 41-50.

TABLE I.- STRUCTURAL DATA AND FLUTTER RESULTS

Wing section (1)	Material	Thickness, percent chord	Full-span structural aspect ratio, A	Distance elastic axis aft of midchord, semichords, a	Distance center of gravity aft of elastic axis, semichords, x_α (2)	Radius of gyration, semichords, r_α (3)	Wing weight per unit span, m lb/ft	Relative density, μ (4)	First natural bending frequency, ω_h, radians/sec (5)	First natural torsion frequency, ω_α, radians/sec (5)	Test-section speed of sound at flutter, c, ft/sec	Flutter Mach number in tunnel, M	Flutter frequency in tunnel ω, radians/sec	Flutter Mach number by standard analysis, M_o (6)	Flutter frequency by standard analysis, ω_o, radians/sec	Flutter Mach number by refined analysis, M_r (6)	Flutter frequency by refined analysis ω_r, radians/sec	$\frac{M}{M_o}$	$\frac{M}{M_r}$
63A002	alum.	2	6	-0.3	0.146	0.478	0.575	43.1	61	421	1106	0.45	----	0.353	208	0.460	157	1.275	0.978
63A002	alum.	2	4	-.3	.146	.478	.575	46.0	137	632	1106	.55	----	.525	316	.610	247	1.047	.902
64A002	alum.	2	3	-.234	.104	.499	.578	56.5	225	828	1097	.80	314	.752	439	.810	306	1.064	.988
64A002	alum.	2	2-2/3	-.234	.104	.499	.578	65.4	278	927	1083	.96	330	.905	506	.926	338	1.061	1.037
64A002	steel	2	4	-.254	.124	.460	1.62	180.1	136.4	634	1085	.94	---	.959	296	.900	178	.981	1.044
64A002	steel	2	3-1/3	-.254	.124	.460	1.62	186.6	184	773	1082	.98	314	1.183	359	.972	208	.828	1.008

[1]All wings were rectangular with semichord b equal to 0.25 feet; all airfoil sections were symmetrical; the models from reference 1 were the NACA 64A series.
[2]Location of center of gravity determined analytically; location of elastic axis measured.
[3]Radius of gyration determined analytically.
[4]Air density, ρ, in mass-density ratio is that in test section at flutter; ρ in pounds per cubic foot.
[5]Structural frequencies determined experimentally for the NACA 64A series, analytically for the NACA 63A series.
[6]Theoretical flutter Mach numbers are based on speed of sound in test section at flutter.

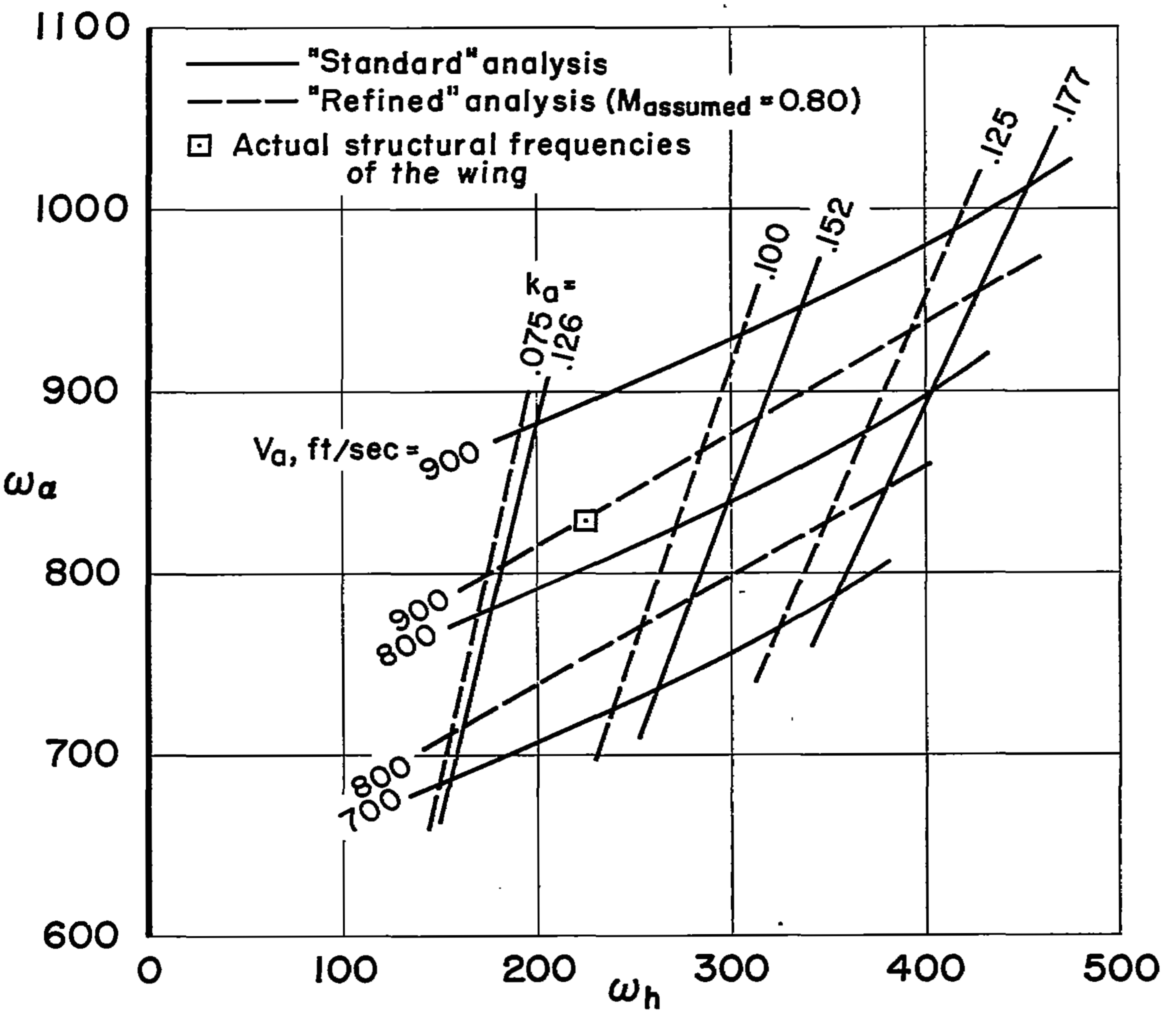


Figure 1.- Parametric flutter curves for the 2-percent-thick aluminum wing of aspect ratio 3.

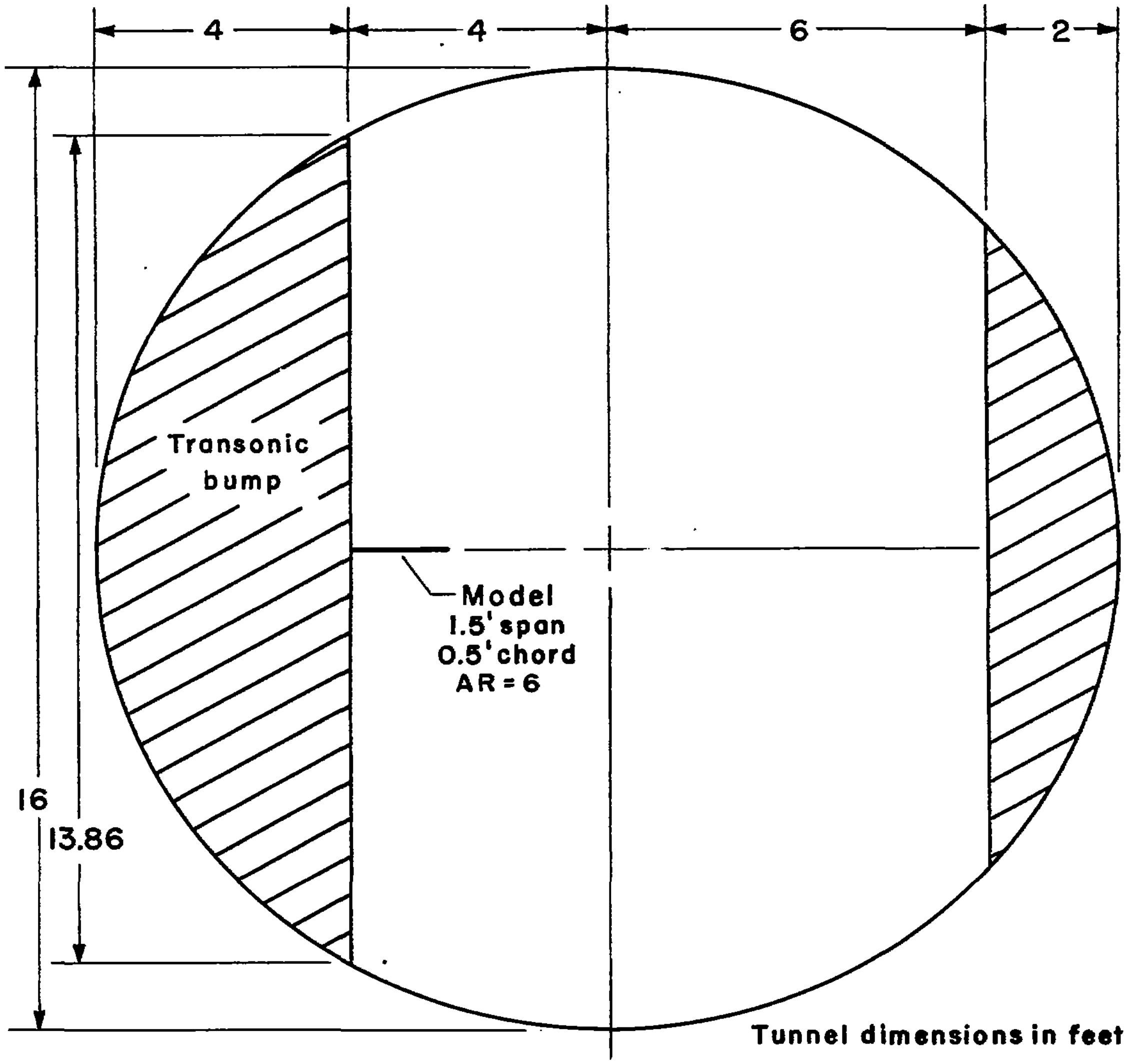


Figure 2.- Sketch of test section showing one of the models in place.

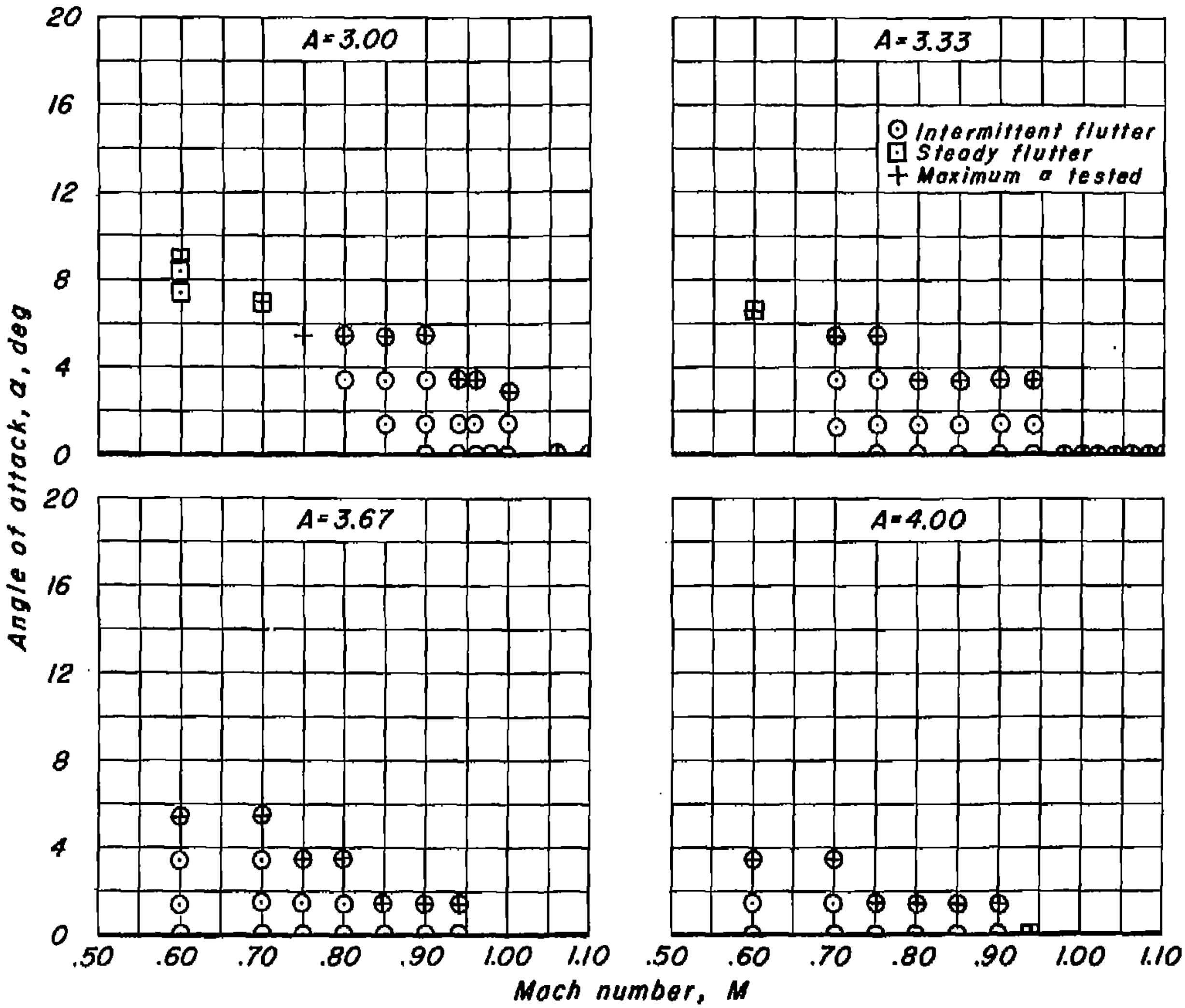


Figure 3.- Flutter observations; steel wing of NACA 64A-002 section.

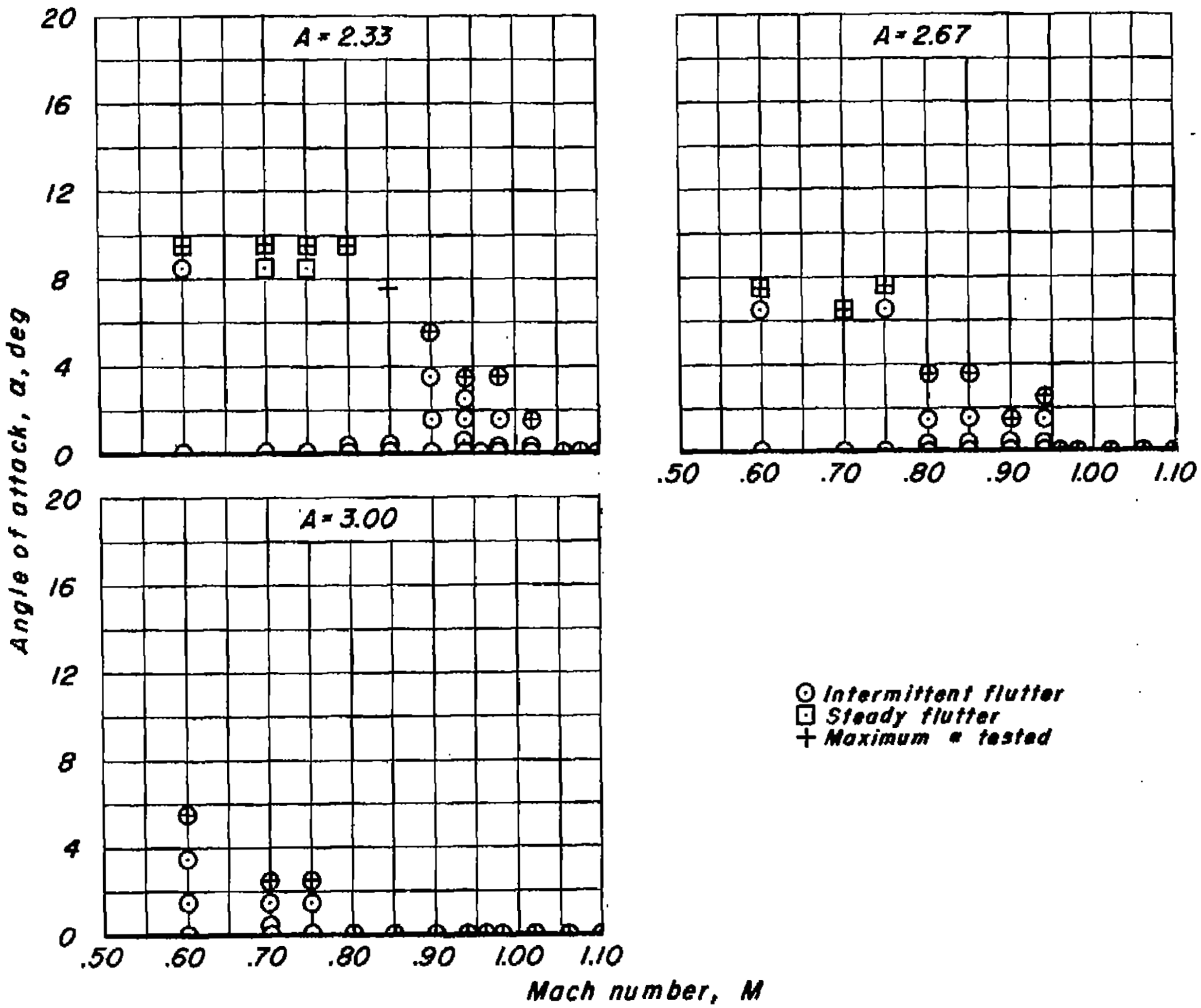


Figure 4.- Flutter observations; aluminum wing of NACA 64A-002 section.

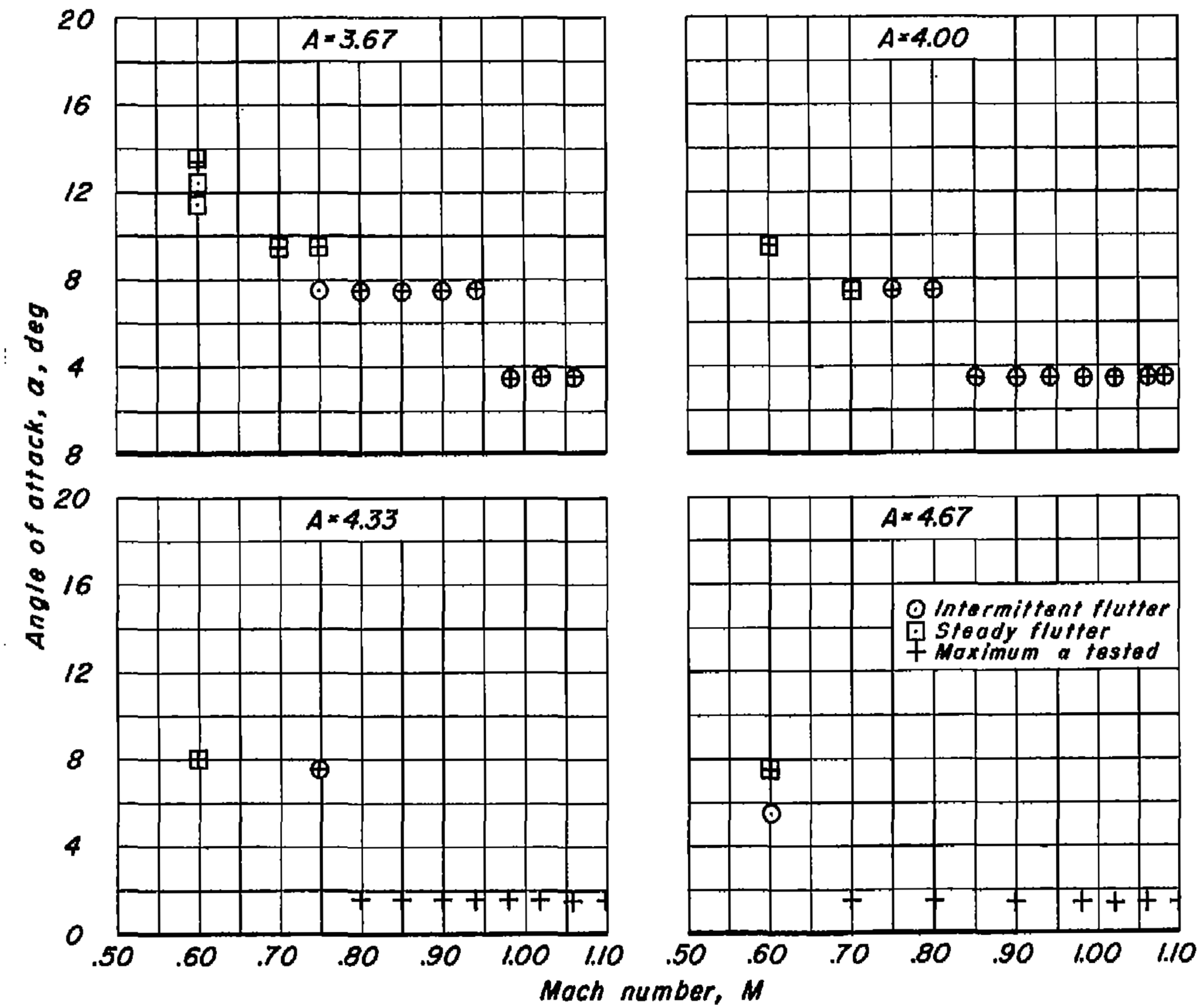


Figure 5.- Flutter observations; aluminum wing of NACA 64A-004 section.

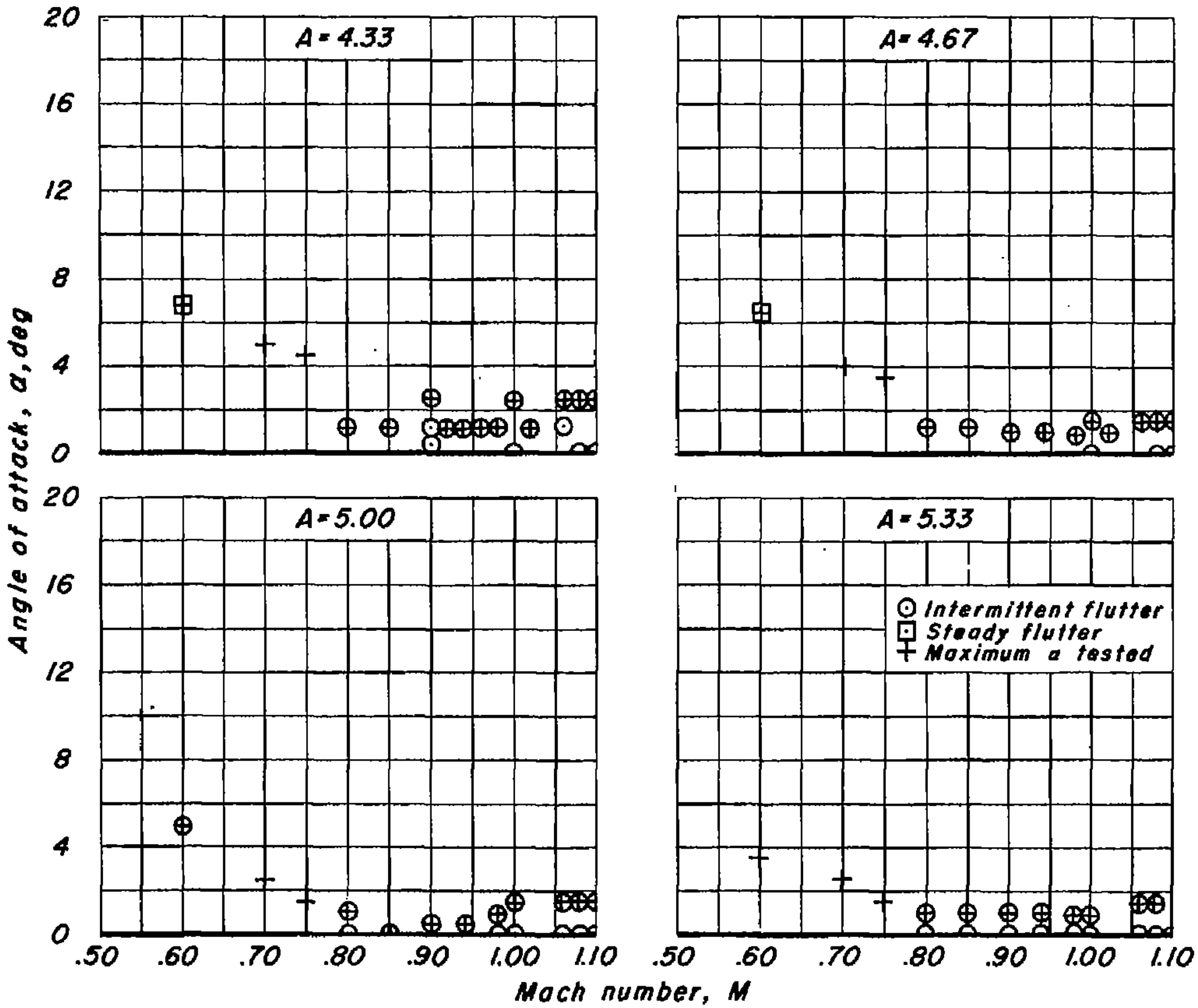


Figure 6.- Flutter observations; aluminum wing of NACA 64A-004 section with spanwise-running slots.

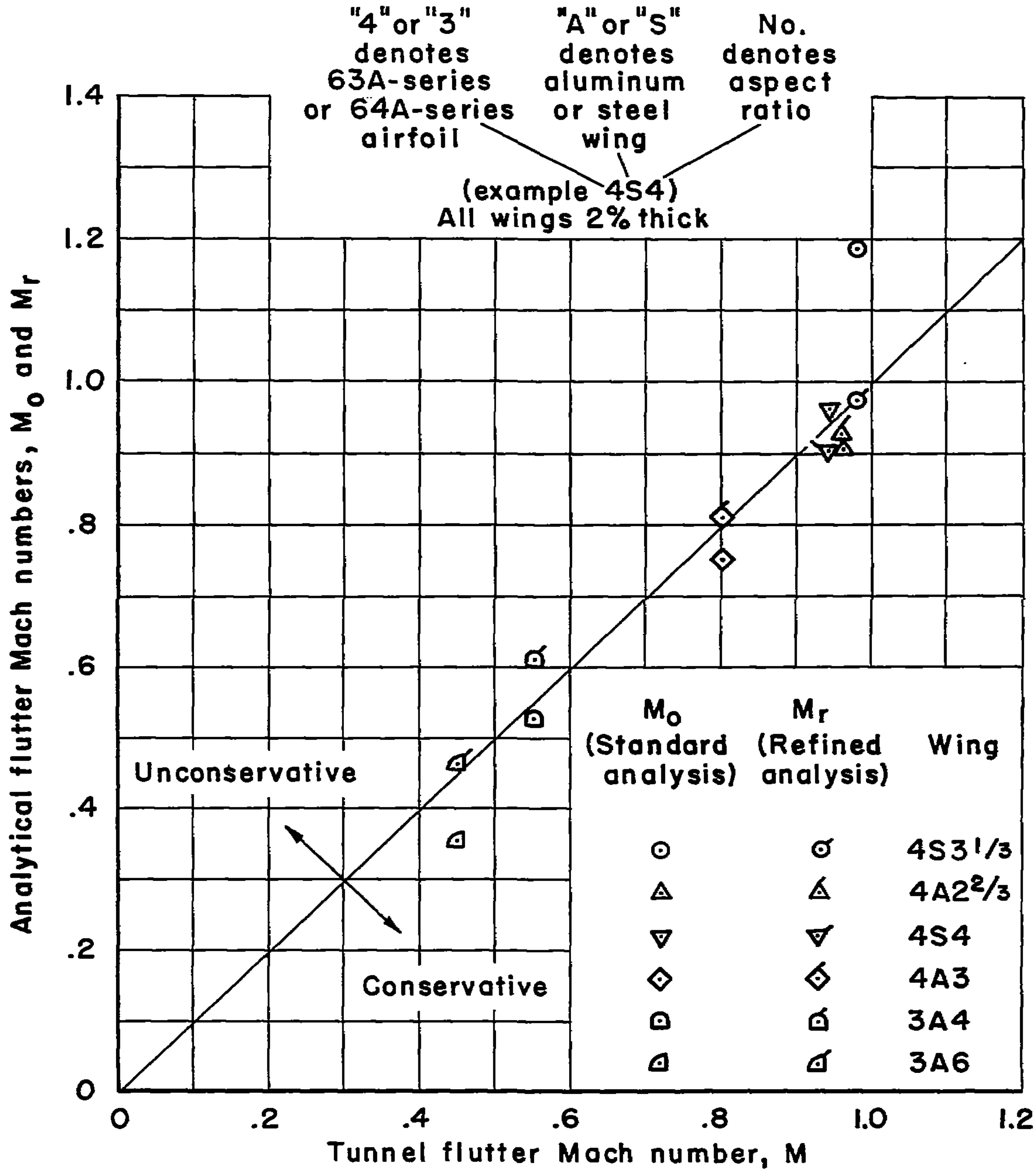
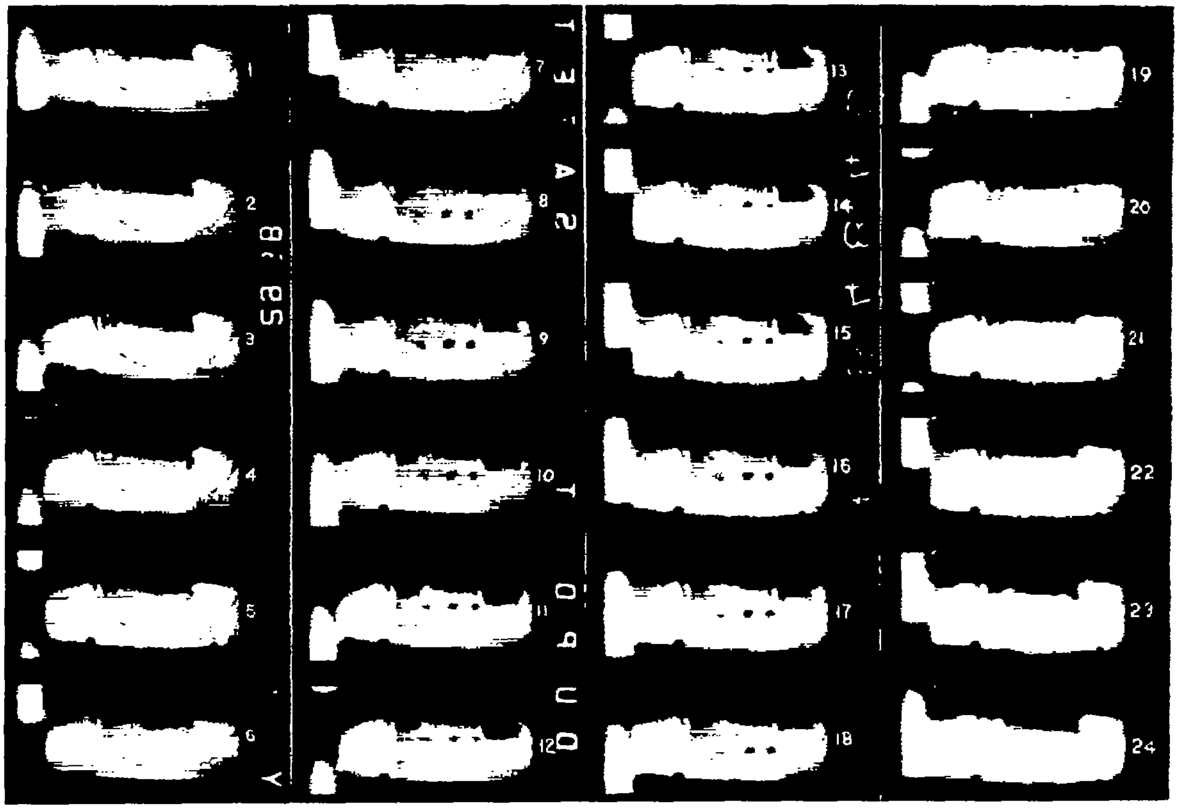


Figure 7.- Comparison of analytical and tunnel flutter Mach numbers.



A-20123

Figure 8.- Cycle of motion of aluminum wing of aspect ratio 3; M = 0.98, $\alpha = 1/2^{\circ}$. (In each frame, trailing edge is on left, leading edge on right.)

[redacted]

[redacted]

[redacted]